An Acoustic Evaluation of Circumferentially Segmented Duct Liners

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An evaluation of circumferentially segmented duct liners is performed by comparing attenuations of optimized segmented and uniform liners for a range of frequencies and source structures. Broadband suppression and the effects of changes to input modal structure are explored in these comparisons. Credence in the theoretical model was obtained by comparing predictions to results of a carefully controlled experiment performed on the Langley Spinning Mode Synthesizer Facility. Excellent agreement was obtained in these comparisons. It is shown that for the lower order spinning mode sources, the optimum segmented liner degenerates into an optimum uniform liner so that no increased suppression of the segmented liner over the uniform liner is obtained. In contrast, predicted results show that the optimized segmented liner is more effective than the optimized uniform liner for the higher order spinning mode sources with a hard-wall/soft-wall admittance variation representing an optimum configuration for the segmented liner. Predicted results also indicate that the segmented liner gives better broadband performance and that its suppression characteristics are not particularly sensitive to changes in input modal structure when compared to that of the uniform liner. It is concluded that the greatest suppression benefit of segmented liners occurs near mode cuton frequencies and that these benefits are obtained despite reductions in the total amount of acoustic treatment of 50% or more.

Nomenclature

A_ℓ, B_ℓ	= amplitude of right and left moving wave in circumferentially segmented liner
$A^\ell_{m,n}, B^\ell_{m,n}$	= coefficients in hard-wall duct expansion of the exact eigenfunction mode, $P_\ell(r,\theta)$
$\bar{a}$	= duct radius
$A^I_{m,n}, B^I_{m,n}$	= coefficients of Bessel-Fourier expansion of incident wave in the first hard-wall section
$A^t_{m,n}, B^t_{m,n}$	= coefficients of Bessel-Fourier expansion of transmitted wave in the second hard-wall section
$\bar{c}$	= ambient speed of sound
i	= unit imaginary number
J_m	= Bessel function of first kind of order m
K	= driving frequency
$\tilde{K}_0$	= tuning frequency
$K_\ell, \Omega_{m,n}$	= axial wave numbers
L	= predetermined length over which transmission loss is computed
m	= circumferential mode orders
m_0	= source circumferential mode order
$N_{m,n}$	= normalization constant for hard-wall basis functions
n	= radial mode orders
n_0	= source radial mode order
$P(r,\theta,z)$	= acoustic pressure field in duct with circumferentially segmented liner
$P^I(r,\theta,z), P^R(r,\theta,z)$	= incident and reflected pressure field in first hard-wall section
$P^t(r,\theta,z)$	= transmitted wave in second hard-wall section
$P_\ell(r,\theta)$	= acoustic pressure eigenfunction
Re( )	= real part of complex expression
r,z	= radial and axial coordinate
T	= periodicity of segmented liner
$TL(\beta_1,\beta_2)$	= total transmission loss function
TL_m	= transmission loss of circumferential mode m
$TL_{m,n}$	= transmission loss of circumferential mode (m,n)
$W(z)$	= total sound power at axial location z
$W_m(z)$	= sound power in mode m,n at axial location z
$\beta(\theta), \beta_1, \beta_2$	= acoustic admittance
$\lambda_{m,n}$	= eigenvalues of hard-wall duct
$\bar{\rho}$	= ambient density
$\delta K(\tilde{K}_0)$	= off-design performance parameter
δK_{ref}	= normalization constant for $\delta K(\tilde{K}_0)$

Subscripts

ℓ	= exact eigenfunction index
m	= circumferential wave number index
n	= radial eigenfunction index for hard-wall duct

Superscripts

$(\bar{\ })$	= dimensional quantity
$(\ )'$	= derivative with respect to the argument

Introduction

WITH the current interest in shorter and less drag producing engine inlets, there is a continuing need for lighter weight, more efficient duct liner treatments for the suppression of turbomachinery noise. Modal conditioning is one method for obtaining improved suppression, especially for discrete tones.[1] Here, the energy of dominant source modes is redistributed into higher order modes which are more easily suppressed by the liner. This modal conditioning is generally achieved by liner nonuniformities. Several investigators[2-6] have investigated the possibility of improved liner performance by means of modal redistribution by incorporating a circumferentially segmented liner into the duct

Received Feb. 26, 1983; presented as Paper 83-0732 at the AIAA Eighth Aeroacoustics Conference, Atlanta, Ga., April 11-14, 1983; revision received July 25, 1983. This paper is declared a work of the U.S. Government and is therefore in the public domain.

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(Fig. 1). A beneficial effect of circumferential segmentation was first observed by Mani,[2] who simply blocked off segments of a uniform liner with strips of aluminum tape. Improved performance of this configuration was ascribed to modal conditioning (redistribution of acoustic energy into higher order circumferential modes which are more rapidly suppressed by the liner). Some analytical studies have followed.[3-6] However, before circumferentially segmented liners can be fully evaluated, additional studies evaluating the effects of frequency, source complexity, duct length, and broadband suppression properties are needed. In addition, carefully controlled experiments are needed to verify these theories.

This paper performs an evaluation of circumferentially segmented duct liners by comparing optimum attenuation characteristics of uniform and circumferentially segmented duct liners. Effects of a broad range of source frequencies and circumferential mode orders are considered. Special consideration is given to broadband attenuation characteristics of the uniform and segmented liner. Finally, to give confidence in the analytical prediction model, results from the prediction are compared to a carefully controlled experiment performed on the Langley Spinning Mode Synthesizer Facility.

Theory

The theoretical results presented in this paper are based on an analysis which is developed in detail elsewhere.[5,6] In this section only the underlying assumptions and enough background information to establish the nomenclature used here are given. However, since the analysis for determining broadband suppression of the two liners was not developed in the above two references, it will be discussed in somewhat more detail.

Consider a circular duct with radius $\bar{a}$, length $\bar{L}$, and a circumferentially segmented duct liner as shown in Fig. 1. The segmented liner is assumed to consist of two piecewise uniform liners with admittances β_1 and β_2 as shown in the figure. These piecewise uniform liners are combined in circumferential series to form a total of $2T$ strips in which T is the periodicity of the segmented lining.

Here we consider the problem of scattering from two axial discontinuities by combining the finite length lining in Fig. 1 with two semi-infinite hard-wall sections as depicted in Fig. 2. In this figure all unbarred quantities are dimensionless, with all distances referred to the duct radius a and acoustic pressures referred to the quantity $\bar{\rho}\bar{c}^2$. As depicted in Fig. 2, an incident sound field on the source side of the liner can be described as a superposition of hard-wall modes at the liner/hard-wall interface as follows:

$$P(r,\theta,z) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[A^I_{m,n}\cos m\theta + B^I_{m,n}\sin m\theta\right] \times \frac{J_m(\lambda_{m,n}r)}{N_{m,n}}\, e^{i\Omega_{m,n}z} \tag{1}$$

Similarly, the transmitted sound on the termination side of the liner is expressed as a superposition of hard-wall modes:

$$P(r,\theta,z) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[A^t_{m,n}\cos m\theta + B^t_{m,n}\sin m\theta\right] \times \frac{J_m(\lambda_{m,n}r)}{N_{m,n}}\, e^{i\Omega_{m,n}z} \tag{2}$$

while the sound field in the circumferentially segmented liner is expressed in terms of the characteristic functions, $P_\ell(r,\theta)$ for the segmented liner:

$$P(r,\theta,z) = \sum_{\ell=0}^{\infty}\left[A_\ell e^{iK_\ell z} + B_\ell e^{-iK_\ell(z-L)}\right]P_\ell(r,\theta) \tag{3}$$

Galerkin's method is employed to obtain the characteristic function, $P_\ell(r,\theta)$ as well as the axial wave number, K_ℓ, for the segmented liner.[5] This modal expansion [Eq. (3)] is then matched to the hard-wall modes at the appropriate interface to determine the reflected sound P^R on the source side of the liner as well as the transmitted sound on the termination side of the liner. The attenuation is

$$TL = 10\log_{10}[W(0)/W(L)]$$

$$W(0) = \sum_{m=0}^{\infty} W_m(0), \qquad W(L) = \sum_{m=0}^{\infty} W_m(L) \tag{4}$$

whereas the attenuation function for circumferential mode m is

$$TL_m = 10\log_{10}[W_m(0)/W_m(L)]$$

$$W_m(0) = \sum_{n=0}^{\infty} W_{m,n}(0), \qquad W_m(L) = \sum_{n=0}^{\infty} W_{m,n}(L) \tag{5}$$

while that for circumferential mode (m,n) is

$$TL_{m,n} = 10\log_{10}\left[\frac{W_{m,n}(0)}{W_{m,n}(L)}\right] \tag{6}$$

where

$$W_{m,n}(L) = (|A^t_{m,n}|^2 + |B^t_{m,n}|^2)\,\mathrm{Re}(\Omega_{m,n}) \tag{7}$$

$$W_{m,n}(0) = (|A^I_{m,n}|^2 + |B^I_{m,n}|^2)\,\mathrm{Re}(\Omega_{m,n}) \tag{8}$$

Further, within the context of this paper, a liner is said to be tuned or optimized at frequency $\tilde{K}_0$ when β_1 and β_2 are chosen to maximize the attenuation function, TL, at that frequency $\tilde{K}_0$.

Comparison of off-design or broadband performance of the uniform and segmented liner is achieved by tuning both liners at the same frequency and comparing their attenuation performance over a frequency range. Let $TL(K;\tilde{K}_0)$ denote the attenuation at frequency K for a liner tuned at frequency $\tilde{K}_0$. Define the parameter $\delta K(\tilde{K}_0)$ as:

$$\delta K(\tilde{K}_0) = \left[\int_{\tilde{K}_0-1}^{\tilde{K}_0+1} TL(K;\tilde{K}_0)\,\mathrm{d}K\right]\Big/\delta K_{\mathrm{ref}} \tag{9}$$

where δK_{ref} is an arbitrary normalization constant. Thus, a plot of the spectrum of $\delta K(\tilde{K}_0)$ allows the off-design performance of a liner to be shown in a more compact form than a plot of the function $TL(K;\tilde{K}_0)$. In this paper, the spectrum of δK is compared for the uniform and segmented liner to compare broadband performance.

Experimental Apparatus

A carefully controlled experiment was designed to validate the analytical predictive program used in this paper. The

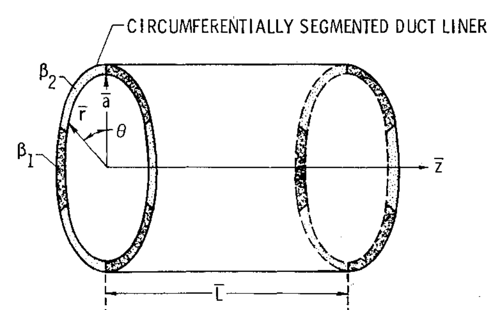

Fig. 1 Schematic of circumferentially segmented duct liner and coordinate system.

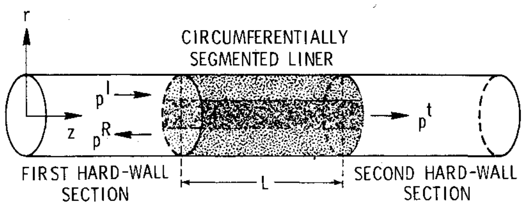

Fig. 2 Configuration employed to develop mode matching equations.

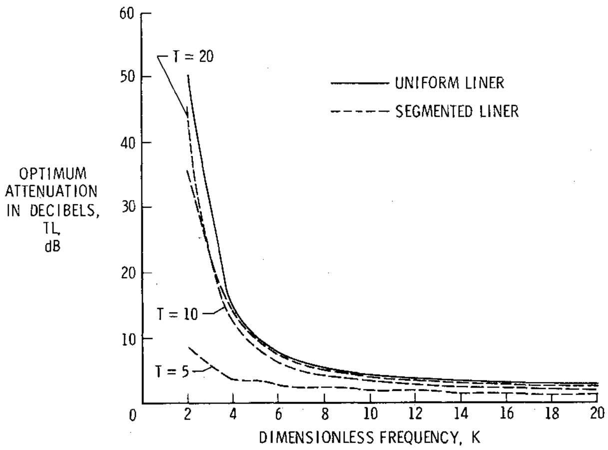

Fig. 3 Comparison of optimum attenuation loss spectra for uniform and segmented liner for plane-wave source. $P^I(r,\theta,0)=1$.

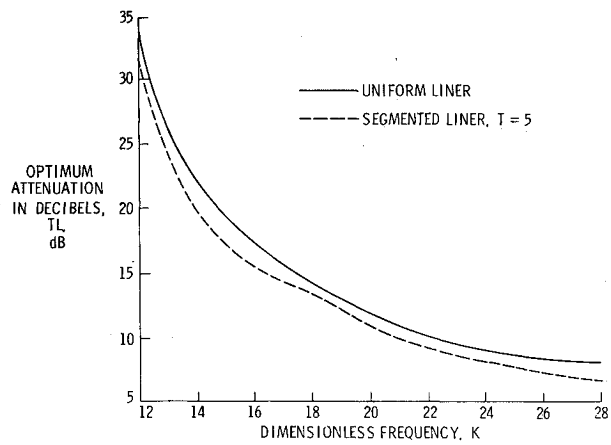

Fig. 4 Comparison of optimum attenuation loss for a 5-lobe source, $P^I(r,\theta,0)=\cos 5\theta J_5(\lambda_{5,0}r)$.

experimental setup for performing this experiment is described in detail in Ref. 7. The length-to-diameter ratio was unity, while the impedance test liner for the experiment was achieved by blocking a 180 deg circumferential segment of the uniform liner with aluminum tape. The admittance of the uniform liner was determined by impedance measurements taken in a standing wave impedance tube. For purposes of comparison with analytical results, two driving frequencies were considered: $K=4.5$, in which the admittance of the specimen was determined to be $\beta=0.295+0.156i$; and $K=7.8$, in which the admittance of the specimen was determined to be $\beta=0.065+0.270i$. At the lower driving frequency the (1,0) mode was the incident mode, while for the higher driving frequency the (3,0) mode was the incident mode. Further details of the spinning mode synthesizer operation are described in Ref. 7.

Results and Discussion

Results for trend studies as well as comparison of theory and experiment are presented. In all cases the uniform liner result is used as a baseline with which to measure the performance of the segmented liner. First, optimization studies for both high and low order circumferential modes are presented. Next, the off-design performance and changes in source input structure are considered. To reduce computational cost, the length-to-diameter ratio in these studies is restricted to 1.0 (i.e., $L=2$). Finally, a comparison between theory and experiment is given.

Optimization Studies

Optimization results are presented in this subsection. The optimization method used here is discussed in some detail in Ref. 6 and will not be repeated here. To determine the effectiveness of the segmented lining, the attenuation spectrum for the lining is compared to that of a uniform lining for a range of operating frequencies. Source inputs considered are a plane wave, a 5-lobe source, and 10-lobe source. First, optimization studies are presented in which one section of the segmented lining is a hardwall ($\beta_1=0$) and the admittance of the second section is employed as the optimization parameter (hard-wall/soft-wall optimization studies). Next, the admittances of both sections are varied in the optimization algorithm (soft-wall/soft-wall optimization studies).

Figures 3, 4, and 5 show the optimum attenuation spectra for a plane wave $[P^I(r,\theta,0)=1]$, 5-lobe source $[P^I(r,\theta,0)=\cos 5\theta J_5(\lambda_{5,0}r)]$, and a 10-lobe source $[P^I(r,\theta,0)=\cos 10\theta J_{10}(\lambda_{10,0},r)]$, respectively. In the three figures the admittance β_1 is restricted to a hardwall. The periodicity of the segmented liner plays an important role for the segmented liner. The transmission loss of the segmented liner is an increasing function of the periodicity at each value of frequency for the plane wave source. However, when compared to the uniform liner, the segmented liner suppresses less sound for the plane wave source. The same is true for the 5-lobe source. In contrast, the segmented liner suppresses more sound than the uniform liner for the 10-lobe source, with the greatest increase in suppression occurring near cut on of the incident mode. This is rather surprising, since the segmented liner uses only 50% as much treatment when compared to the uniform liner (i.e., $\beta_1=0.0i$). It should be noted that the optimum value of β_2 was nearly identical to those obtained in Ref. 6 and can be obtained from that work for each of the sources considered.

Since the increased suppression of the segmented liner over the uniform liner for the 10-lobe source was actually achieved with only one-half the acoustic treatment of the uniform liner, it was thought that an even greater suppression might be obtained for a fully soft segmented liner. Results of the soft-wall/soft-wall optimization studies were identical to the uniform liner results for the plane and 5-lobe source with $\beta_1=\beta_2$ at the optimum. In addition, the results show that for the 10-lobe source, the optimum occurs when $\beta_1=0.0i$. Thus, a hard-wall/soft-wall admittance variation represents the best lining configuration for a 10-lobe source.

Off-Design Performance

Comparison of off-design performance of two liners is achieved by tuning both liners at the same frequency and comparing their attenuation performance over a frequency range. Figures 6 and 7 compare $TL(K;\tilde{K}_0)$ for a uniform and segmented liner where the tuning frequency $\tilde{K}_0=17$ and 19, respectively, and the source is a 10-lobe source. Thus, for these tuning frequencies the segmented liner is observed to give better off-design performance over a broader frequency range than the uniform liner. Further, when the tuning frequencies were changed to 18, 20, 21, and 22 similar trends were obtained.

Figure 8 shows a plot of $\delta K(\tilde{K}_0)$ for the uniform and segmented liner. Here the source is a 10-lobe source and the function $\delta K(\tilde{K}_0)$ is normalized so as to keep the magnitude of the function plotted along the transverse axis less than unity

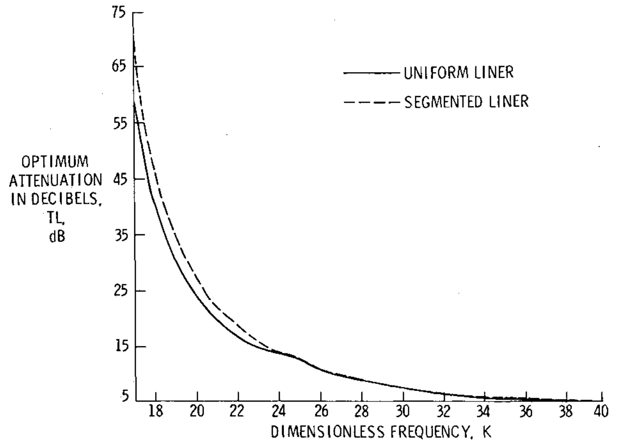

Fig. 5 Comparison of optimum attenuation spectrum for a uniform and segmented liner for a 10-lobe source, $P^I(r,\theta,0) = \cos 10\theta J_{10}(\lambda_{10,0} r)$.

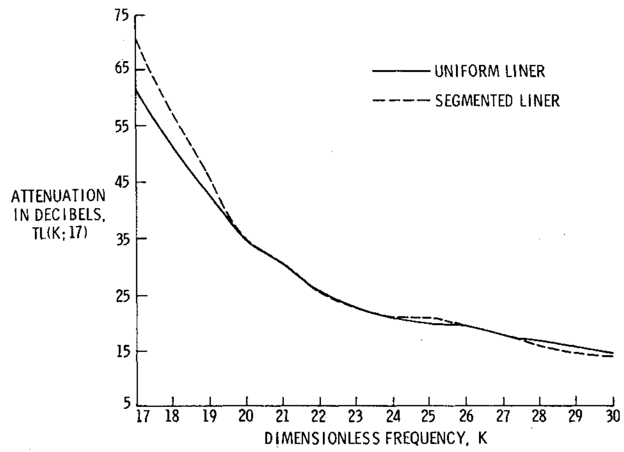

Fig. 6 Comparison of the off-design performance for a uniform and segmented liner tuned at 17 for a 10-lobe source.

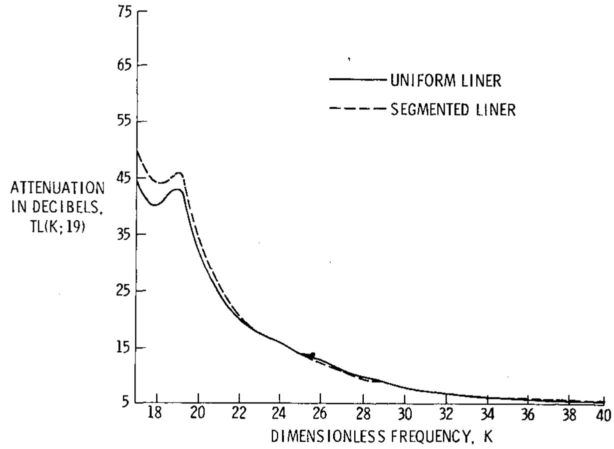

Fig. 7 Comparison of the off-design performance for a uniform and segmented liner tuned at 19 for a 10-lobe source.

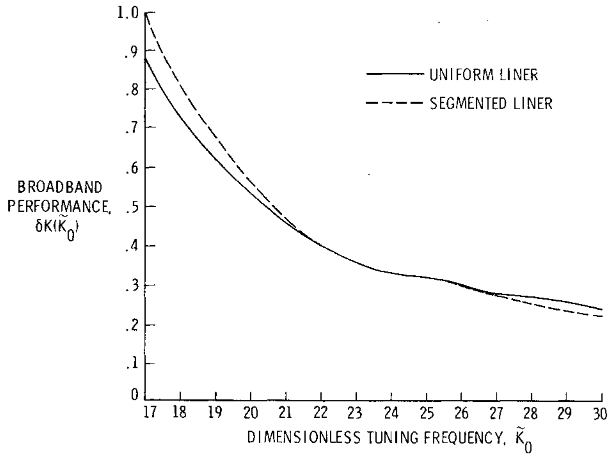

Fig. 8 Comparison of broadband performance for a uniform and segmented liner for a 10-lobe source.

so that

$$\delta K_{\text{ref}} = \int_{16}^{18} TL(K;17)\, \mathrm{d}K$$

Note that for $\tilde{K}_0 < 22$, $\delta K(\tilde{K}_0)$ is greater for the segmented liner than for the uniform liner, with the inverse being true for $\tilde{K}_0 > 26$. This shows that the evaluation given in Eq. (9) and plotted in Fig. 8 is consistent with the spectra shown in Figs. 6 and 7. Thus, for tuning frequencies less than 22 the segmented liner gives better off-design performance than the uniform liner, whereas the opposite is true for tuning frequencies greater than 26. It should be noted that although the absolute magnitude of $\delta K(\tilde{K}_0)$ for the uniform liner is greater for $\tilde{K}_0 > 26$, the actual suppressions obtained from the two liners are nearly equal above this tuning frequency.

Changes in Source Structure

To investigate the effects of changes in source structure on the optimum attenuation, the incident wave was distorted so as to contain higher order radial mode content for both the uniform and segmented liner. Thus, the incident wave was changed to

$$P^I(r,\theta,0) = \sum_{n=0}^{9} \cos 10\theta J_{10}(\lambda_{10,n} r)$$

and the attenuation spectrum for both liners recomputed with all parameters the same as in Fig. 8. Results are presented in Fig. 9. Note that the segmented liner continues to give better performance than the uniform liner for dimensionless frequencies up to 22. For dimensionless frequencies beyond this value the uniform liner gives slightly more suppression.

Comparison of Theory and Experiment

To provide perspective, the results of the experiment performed on the spinning mode synthesizer facility will now be compared to analytical predictions for the two test frequencies, $K = 4.5$ and 7.8. Comparisons will be presented for both the uniform liner and a segmented liner constructed by covering a 180 deg segment of the uniform liner with tape. Only a single spinning mode (m,n) is allowed as source input. Such comparisons are very useful in that they allow validation of the theoretical prediction program and lead to a degree of confidence in the conclusions.

Table 1 represents typical measured and predicted attenuation levels TL_{m0} of the incident spinning mode for the two test frequencies investigated in the experiment for the uniform liner. Since all mode levels were referenced to that of the dominant mode, the transmitted mode levels can be considered as modal attenuations. It should be noted that the upper limit on modal isolations achieved in this experiment is a system dynamic range determined jointly by the spinning mode synthesizer and the data acquisition system. Typical contributors to the mode isolation noise floor are random calibration errors, random probe positioning errors, and electronic noise. Therefore in view of the foregoing remarks,

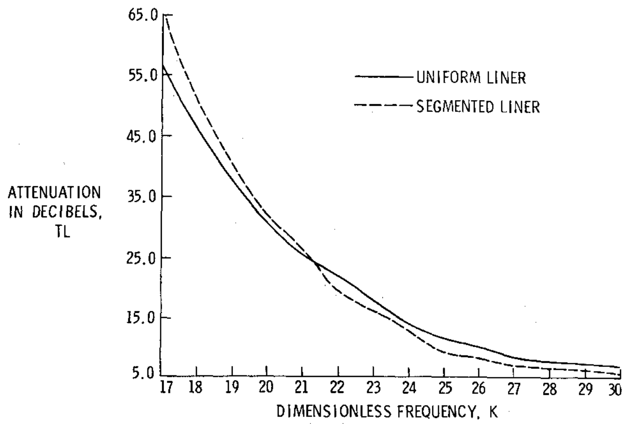

Fig. 9 Effects of changes in source structure on an optimum uniform and segmented liner for a 10-lobe source.

Table 1 Comparison of measured and predicted transmission loss levels for a uniform liner

Dimensionless frequency, K	Incident mode order (m_0, n_0)	Measured transmission loss, dB TL_{m_0}	Predicted transmission loss, dB TL_{m_0}
4.5	(1,0)	10.1	9.9
7.8	(3,0)	5.5	5.2

Table 2 Comparison of measured and predicted transmission loss levels for a two-segment liner with (1,0) mode incident (K = 4.5)

Mode order, (m,n)	Measured transmission loss, dB, $TL_{m,n}$	Predicted transmission loss, dB, $TL_{m,n}$
(0,0)	29.0	10.4
(1,0)	4.0	3.9
(2,0)	9.0	9.8
(3,0)	17.8	16.0

Table 3 Comparison of measured and predicted transmission loss levels for a two-segment liner with (3,0) mode incident (K = 7.8)

Mode order, (m,n)	Measured transmission loss, dB, $TL_{m,n}$	Predicted transmission loss, dB, $TL_{m,n}$
(1,0)	17.5	25.0
(2,0)	8.0	11.0
(3,0)	2.5	2.9
(4,0)	9.8	10.0

only data from 0 to −20 dB in the experiment will be taken as quantitatively meaningful. However, mode levels below these values are shown in the belief that collective trends may be meaningful in a qualitative sense. Excellent comparison between theory and experiment is obtained for these test frequencies, indicating that the theoretical prediction and experimental measurement are fundamentally sound. Furthermore, there are no additional spinning modes generated by the uniform liner than are present at the source. Thus, uniform liners are incapable of scattering acoustic energy among spinning modes of different orders.

Results for the two-segment liner at test frequency 4.5 are shown in Table 2. For the two-segment test liner, significant modal redistributions are seen to occur into the (2,0) mode when driven by dominant mode (1,0) at $K = 4.5$. Further, some scattering has occurred into the (3,0) mode for this test frequency. Excellent comparisons between theory and experiment are obtained for the (1,0), (2,0), and (3,0) modes. In contrast, the analytical model shows the (0,0) mode at 10.4 dB, whereas the measurement shows this mode well down into the noise floor at 29 dB. Note also that this $m = 0$ circumferential mode is the only circumferential mode with higher order radial cuton [i.e., the (0,1) mode is also cut on at this test frequency].

Results for the two-segment liner at test frequency 7.8 are also shown in Table 3. Here, significant scattering into the (1,0), (2,0), and (4,0) modes is observed when the dominant source mode is the (3,0) mode. Excellent agreement between theory and experiment is obtained for the circumferential modes which have no higher order radial modes cut on (i.e., (4,0) and (3,0) modes. Further, there is good agreement between theory and experiment for the (2,0) mode (where the first order radial is barely cut on). There is poor agreement for the (1,0) mode (where the $m = 1$ circumferential mode has two radial modes cut on at this frequency).

The cases depicted in Tables 2 and 3 offer reliable evidence of the ability of the segmented liner to scatter acoustic energy into circumferential modes of different orders. This is a property which is expected to enhance their suppression capability. Furthermore, these cases show that comparisons between theory and experiment deteriorate for the (m,n)th mode when the frequency is such that multiple radial modes are cut on for the mth circumferential mode.

Conclusions

Based upon the results of this work, the following conclusions are drawn:

1) The theoretical predictive model developed in Refs. 5 and 6 agrees well with experimental data.

2) Circumferentially segmented liners scatter acoustic energy into circumferential modes of different orders.

3) For lower order spinning mode sources the optimum segmented liner degenerates into an optimum uniform liner, so that no increased suppression of the segmented liner over the uniform liner is obtained.

4) For the higher order spinning mode sources the optimum segmented liner occurs when one of the segments is a hard wall and gives as much as 7 dB more suppression than the uniform liner near cut on.

5) The segmented liner gives better broadband performance than the uniform liner and is not particularly sensitive to changes in modal structure of the source.

6) From a practical standpoint, the greatest usefulness of circumferentially segmented duct liners lies in their ability to attenuate more sound than uniform liners, with at least a 50% reduction in the amount of treatment.

It should be noted that these conclusions may not be true generally since they were obtained for a length-to-diameter ratio of unity, a single spinning mode source, and no flow through the duct.

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